Differential Equations, Fall 2019
Eviction Project

Name: ______________________________

1 Project Description

This project is based on a project written by Mary Vanderschoot. https://www.simiode.org/resources/4872

2 STATEMENT

According to the National Law Center on Homelessness and Poverty, unaffordable rents and a lack of legal protections for renters have created a national "eviction epidemic" [?]. Matthew Desmond, author of *Evicted: Poverty and Profit in the American City* and director of the Eviction Lab at Princeton University, estimates that 2.3 million evictions were filed in the U.S. in 2016 (four evictions per minute). Desmond writes, "Eviction is a direct cause of homelessness, but it also is a cause of residential instability, school instability [and] community instability" [?]. In this project you will develop and analyze two mathematical models to study eviction trends in a city using an actual eviction rate.

3 Part I: A Linear Model

Suppose a certain city has 118,000 non-homeowner households and this number remains constant each year. (For example, if three of these households move to a different city or purchase a home, then three new non-homeowner households move into this city.) Furthermore, suppose that each of these households is either renting an apartment or house or is not renting due to having been evicted. We can define each of these subpopulations as functions of time, t, (years) in the following manner:

$R(t)$ is the number of *renting* households at time t,
$E(t)$ is the number of *evicted* households at time t.

In order to simplify our calculations, we will consider the fraction of households in each category, that is, if N is the total number of non-homeowner households (in our example $N = 118,000$), then

$r(t) = R(t)/N$ is the fraction of *renting* households at time t,
$e(t) = E(t)/N$ is the fraction of *evicted* households at time t.

The Eviction Lab at Princeton University has developed a nationwide database of eviction records, which includes state and city eviction rates [?]. (For example, in 2016 North Charleston, South Carolina, had the highest eviction rate of 16.5%.) In our model we will assume that a fixed percent α from the renting group become evicted each year. (For North Charleston we would set $\alpha = 0.165$.) We will also assume that a fixed percent β of the evicted group become renters each year. In this model, the only way a renting household can leave the renting group is by transitioning to the evicted group. Similarly, the only way an evicted household can leave the evicted group is by transitioning to the renting group. It is helpful to represent this scenario with a flow diagram in Figure ??.

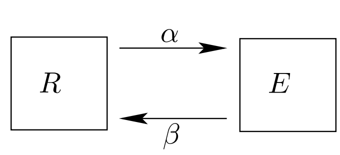

Figure 1: Flow diagram for evictiobn model.

Part of your write up will be by hand on this handout. Part will be in a Matlab mlx file.
Guidelines for Matlab file:

- Start a new matlab file. You are welcome to use any of the code I have provided, but cut and paste it as needed into your file. You should not need to use all the code: just use the code you need to answer the questions and/or justify your answers.
- Clearly label each part of the matlab code which question it addresses. Please use a text box to do this.
- Do not submit any code that does not run.
- Please make sure all the output is showing when you submit the file to canvas. I'll rerun any code I need to, but having the output already showing helps a great deal. Thank you.

The following will guide you through the lab. Provide answers as you go on this handout:

1. Find equations for $\frac{dr}{dt}$ and $\frac{de}{dt}$ that satisfy the above assumptions. Explain the meanings of $\frac{dr}{dt}$ and $\frac{de}{dt}$ and what each component of your equations represents.

2. Choose a value for α from the 2016 data for city evictions from the Eviction Lab [?]. (State which city's information you chose.) Set $\beta = \frac{1}{3}\alpha$. What does the assumption, $\beta = \frac{1}{3}\alpha$ mean in the context of this scenario?

3. Sketch the phase portrait for this system, including all equilibrium solutions. (Be sure to sketch only the region relevant to our situation.) Provide this phase portrait in the Mablab file. Indicate here what the equilibrium solutions are. (Hint: look at the eigenvectors.)

4. For our model, we are only concerned with initial conditions on the line $r + e = 1$. Why? Sketch this line on your phase portrait. You can add this line to your phase portrait by defining a function $g(s) = 1 - s$ (just use letters you haven't used somewhere else) and plotting its graph. Maybe make the line a different color so it stands out. Explain here why we are only interested in the line $r+e = 1$.

5. Suppose that, initially, 95% of non-homeowner households are renters (so 5% are in the evicted group). Solve this initial value problem and plot $r(t)$ and $e(t)$ on one graph. What does this model predict about the percent of non-homeowner households in each group (renting and evicted) in the long-run? Eventually, what will be the (approximate) ratio of renting to evicted non-homeowner households?

6. How might this model be overly simplistic? What are some additional considerations that should be included in an eviction model? Give at least two answers. You might suggest modification to the model to correspond to your additional considerations.

7. Consider the following statement:

   "A city with a higher eviction rate will have a higher percentage of people in the *evicted* category at any given time."

   If this statement is true, provide a mathematical justification that it is true.

   If this statement is false, provide a mathematical explanation for why it is false. If the statement is false, what role does the eviction rate play in this mathematical model?

4 PART II: A Modified Model

In our first model we assumed that a constant fraction of the renting group, αr, transitioned to the evicted group each year regardless of the vacancy rate. Suppose that the city has exactly M rental units (apartments and houses).

In this section you will construct a model for $\frac{dR}{dt}$ and $\frac{dE}{dt}$ that satisfies the following assumptions:

- The flow rate from the renting group to the evicted group increases as the number of vacancies decrease (i.e. when R is close to M).
- The flow rate from the evicted group to the renting group decreases as the number of vacancies decrease.

Notice!: Use R and E in this part, NOT the percentages r and e. ALSO: you need to read the two assumptions above carefully. I read them about 8 times.

1. First, I suggest you copy ALL the code you used for the first part of this lab and paste a copy at the end of your file. You will probably be able to modify the code to answer this second part. Do NOT overwrite what you did for the first part!

2. Suppose your roommate suggests using $-\alpha\frac{R^2}{M}$ for the flow rate from the renting group to the evicted group. Does your roommate's suggested flow rate satisfy the first assumption? If so, explain why it does; if not, suggest a different formula and show that it satisfies the first assumption.

   (Hint: if you write $-\alpha\frac{R^2}{M} = -\alpha\frac{R}{M}R$ it might help to think about what's going on.). You are welcome to suggest a different way to modify the model, just keep in mind that more complicated is not necessarily better.

3. Find a modification that can be used for the flow rate from the evicted group to the renting group. Explain why your modification satisfies the second assumption. The idea is to replace the $+\beta E$ term with something that satisfies the second assumption. Just as in the previous modification, you'll want to multiply βE by something. Something that is small (close to 0) when there are few vacancies and large (close to 1) when there are many vacancies. Write your proposed modification here.

4. Write the system of differential equations in R and E that represent the modification of the model.

5. Suppose that $M = 118,500$. Using the same values of α and β from Part I, use technology to sketch the phase plane and the solution curves that satisfy the initial conditions $R(0) = 112,100$ and $E(0) = 5900$ (so initially, 95% of the non-homeowners are renting). What does this model predict about the long term percentages of non-homeowner households who are renting and who are evicted? How does this compare to your results in the first model?

   Carefully modify your matlab code. Show the two plots: a phase plane and a time plot. If you used `ode45`, you can access the final entry in the numerical solution with: `xa(length(xa),1)` and `xa(length(xa), 2)`. For comparison purposes compute the percentage of renters and the percentage of evicted.