Differential Equations, Fall 2019
Bifurcation Project

Name: ____________________________________

1 Project Description

1. The point of this project is to develop a "bifurcation diagram" for a system of linear differential equations.

   Consider the system below:

$$\frac{dx}{dt} = \frac{3}{\mu}x + y$$
$$\frac{dy}{dt} = -x + \mu y$$

   (a) Remember the first step to finding a solution to a linear system is to find the eigenvalues. Do that here. Your answer will be in terms of μ.

   (b) There are 3 possibilities for eigenvalues: two real eigenvalues, one repeated real eigenvalue, complex conjugate eigenvalues. Determine the values of μ, or ranges of values of μ for these three cases. Record your answer in the table below.

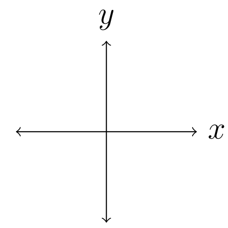
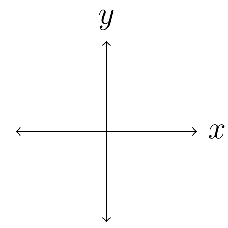
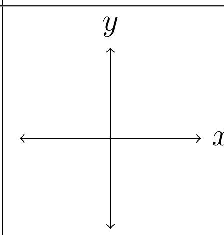
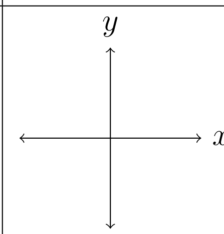
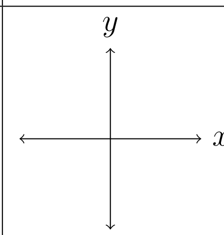
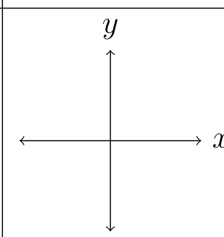
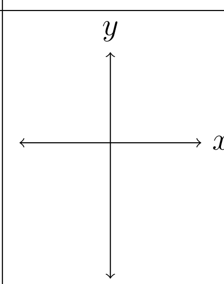
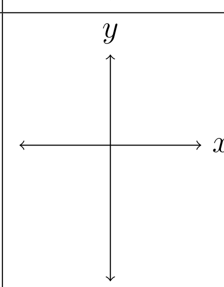
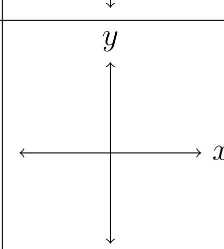
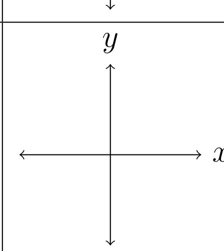
	μ values or range of μ values
two real eigenvalues	
one repeated real eigenvalue	
complex conjugate eigenvalues	

(c) You should have found 4 *endpoints* above. Let's add $\mu = 0$ to the list because clearly something interesting happens to the system when $\mu = 0$. We'll call these 5 values *bifurcation* points because these are the places where the nature of the solution will change. (I'll be honest: I don't know if this is standard terminology.)

(d) Now fill in the chart on the next page.

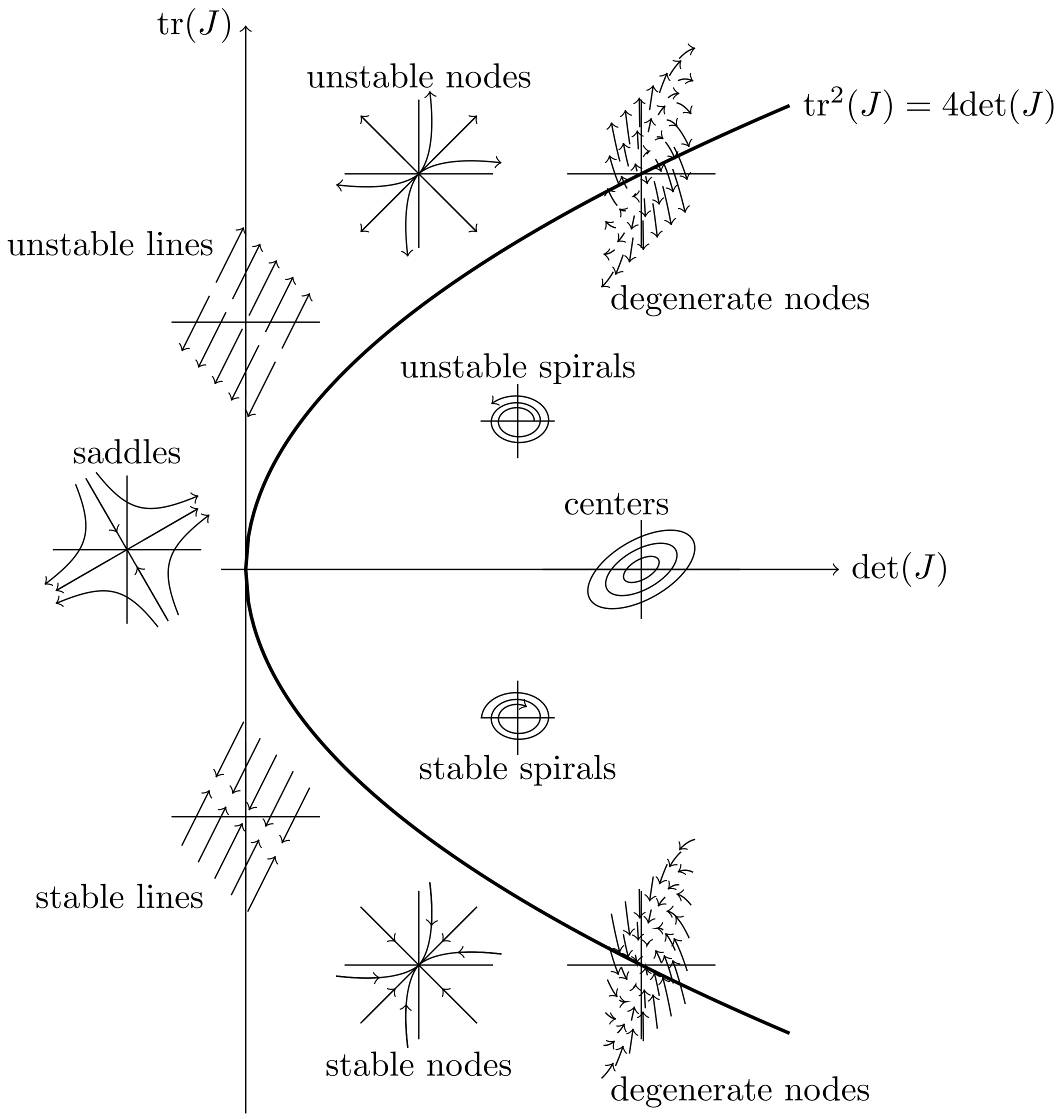
- List your 4 μ-bifurcation values in the boxes with an *.
- Pick (nice) values on either side of the bifurcation values.
- Use Matlab to compute the eigenvalues and eigenvectors. Describe these in the appropriate columns.
- For eigenvalues: state whether there are 2 real, 1 repeated real, or 2 complex. Also, say whether the real part(s) are both positive, both negative, or one of each.
- For eigenvectors: give these: approximations are good enough. For example if Matlab gives $\begin{pmatrix} -0.9456 \\ -0.3254 \end{pmatrix}$, then $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$ is a vector that points in (about) the same direction and is a lot easier to think about.
- In the sketch column: sketch a phase portrait. You can use Matlab to do this, but pplane is easier. If there are real eigenvector(s), show these. Your diagram should show arrow directions: in/out, clockwise/counterclockwise, etc.
- Once you've completed the chart, it describes how the behavior of the solutions to the system change as μ changes.

2 Bifurcation Table

μ	description of eigenvalues	description of eigenvectors	sketch
			
*			
			
*			
			
*$\mu = 0$			
			
*			
			
*			
			

3 Trace-Determinant Diagram

The following is called a *trace-determinant* diagram. (copied from Herman's textbook) If you're curious, you can figure out the connection between this and what you just did. You don't need this diagram to do the exercise and you don't need to do anything on this page.



If $J = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $\mathrm{tr}(J) = a + d$ (the sum of the main diagonal entries), and $\det(J) = ad - bc$.